Anxiety and Performance on Mathematical Problem Solving of Secondary Two Students in Singapore

Yeo Kai Kow, Joseph

National Institute of Education, Nanyang Technological University, Singapore

Abstract: Researchers have highlighted the significance of affective factors and their influences on problem-solving performance. Moreover, it was been reported that research on the cognitive processes in mathematical problem solving has been over-represented resulting in a neglect of non-cognitive factors such as emotions and anxiety. Therefore, the purpose of this study was to identify interrelationships among mathematics anxiety, test anxiety and problem-solving performance. A total of 621 Secondary 2 students from ten secondary schools in Singapore participated in the study. The data showed a positive relationship between mathematics anxiety and test anxiety. In addition, mathematics anxiety and performance on a non-routine mathematical problem solving test showed a marginal linear relationship whereas test anxiety had almost no relationship with the performance on the non-routine mathematical problem solving test.

Introduction

Research interest in mathematical problem solving should be drawn to non-cognitive and affective variables. This view was supported by Southwell (1999) who indicated that "problem solving, then,…involves not only a cognitive element but also an affective element" (p. 332). It has been recognised that some students experience anxiety or other negative emotions associated with mathematics (Buxton, 1984; Hembree, 1990; Ma, 1999) and with testing situations (Hembree, 1987) which are generally regarded as separate but related phenomena (Kazelskis et al., 2000). Further, according to some studies (Mandler, 1989; McLeod, 1989), for students who work on non-routine problems over an extended period of time, the emotional responses frequently become intense. The emotional reaction is described as fear, anxiety, embarrassment and panic (McLeod). This point was also reported in one of the Singapore's newspaper, Straits Times, about primary school students taking the local national examination:

> When PSLE (Primary School Leaving Examination) students, who had done endless assessment papers to prepare for the important examination, came face to face with an unfamiliar scenario, there was panic. (Yeong, 2000, p. 61)

Although Singapore was the top-performing country at Secondary 2 (13 to 14 year old students) for the Third International Mathematics and Science Study (TIMSS) and the Third International Mathematics and Science Study – Repeat (TIMSS-R) in 1995 and 1999 respectively, little research is available on mathematical problem solving and its relationship to mathematics anxiety. In addition, in a recent study by Singapore's Ministry of Education (2000) on a study of pupil stress, it was found that mathematics was perceived to be the most stressful subject to all Secondary 2 students from all courses. With the scarcity of local research in this area, the contribution of the present research is thus significant in providing local data that are pertinent in documenting the relationships among mathematical problem solving, mathematics anxiety, and test anxiety of Secondary 2 students in Singapore. Therefore, the purpose of this research was to identify interrelationships among mathematics anxiety, test anxiety, and problem-solving performance of these students.

Research Literature

Buxton (1981), who had made an important contribution to refining the concept of mathematics anxiety believed in the inseparability of cognition and emotion. She commented that "we operate well or badly in learning and more especially in problem solving, according to the drive provided by our emotions" (p. 3). Researchers interested in the teaching of problem solving have stressed the significance of affective factors (e.g., Lester, 1983) and their influence on problem-solving performance. Silver (1985), in an influential analysis of mathematical problem solving, suggested a number of areas in which more research was needed. He had identified affect as one of the more important underrepresented themes in this research area. Research on the cognitive processes in mathematical problem solving had been over-emphasised resulting in a neglect of non-cognitive factors such as emotions and anxiety (McLeod, 1989). Schoenfeld (1989) also believed that an individual's performance on a mathematical problem was greatly affected by a range of affective factors, sometimes to the extent of controlling the individual's thinking and actions. Traits often associated with successful problem solving were confidence, lack of anxiety, flexibility, lack of rigidity and an ability to cope with uncertainty (Trimmer, 1974).

Several studies on anxiety and mathematical problem solving showed varied results. Mathivanan (1992) conducted a study on a group of 100 Secondary 3 male students. It was found that there was no significant relationship between anxiety towards mathematics and problem-solving achievement. Yet in a study on behavioral traits in mathematical problem solving, Foong (1992) concluded that affective responses tended to distract rather than enhance successful problem solving. She found that many of the unsuccessful problem solvers' initial perceptions of the problem as

difficult aroused a certain level of anxiety and confusion, even before they set to solve the problem. Subsequently when their attempts at solutions did not produce results, they became frustrated and showed lack of self-confidence. They would either continue in a confused manner or they would give up, thus avoiding further frustration. Further, Yudariah and David (1999) found that a majority of the 44 undergraduate students in their study had negative attitudes such as anxiety, fear of new problems and lack of confidence before a problem-solving course. However, after a problem-solving course, the changes in anxiety were in the desired direction. In another study (Tay, 2001) on 62 Secondary 3 high achieving students in Malaysia, it was found that mathematics anxiety existed even among "smart" students. Results indicated that 70% of the students exhibited great anxiety for test items, and that boys were more anxious than girls when dealing with mathematical problem solving. These varied results support the need for further research.

Research Methodology

Subjects

As the academic and mathematical performance varies among secondary schools in Singapore, grouping the secondary schools into homogeneous strata was considered desirable for selecting a representative sample. Although the sample was an opportunity sample, it was deliberately stratified to include the various school types in Singapore. Thus intact class groups were used to include students from all 'selected' schools. A total of 621 Secondary 2 Express or Special[1] students from ten secondary schools participated in this study. All the students would have undergone the same mathematics curriculum. Moreover, based on their mathematics PSLE results, all the students in the sample had passed their examinations. That is, they had either achieved a grade of A*, A, B or C in mathematics. In addition, although problem solving is an integral part of the primary and secondary school program, these students had not been exposed to any explicit mathematics problem-solving instructional program in school.

Instrumentation

Three paper-and-pencil instruments were adapted and used in the study:
- Fennema-Sherman Mathematics Anxiety Scale (MAS), comprising 12 items,
- Test Anxiety Inventory (TAI), comprising 20 items, and
- Problems Test, comprising five problems.

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[1] In Singapore, students who pass the Primary School Leaving Examination (PSLE), after six years of primary school, are streamed according to ability. The academically-able students follow a four-year programme either in the Special or the Express stream.

The Fennema-Sherman Mathematics Anxiety Scale was specifically designed for secondary school students and had a high reliability in the pilot sample. Furthermore, as this scale had been used in Singapore by three researchers (Foong, 1984; Tan, 1990; Lenden-Hitchcock, 1994), it was decided that Fennema-Sherman Mathematics Anxiety Scale (MAS) would be the instrument adopted for assessing mathematics anxiety in the study design. Mathematics Anxiety in this study referred to feelings of anxiety, dread, and nervousness, and associated bodily symptoms related to doing mathematics. A student's mathematics anxiety score can range from 12 to 60. A high total score in the scale would reflect a high level of reported mathematics anxiety whereas a low total score would mean a low level of reported mathematics anxiety.

The Test Anxiety Inventory (TAI) used in this study is a self-report psychometric instrument developed to measure individual differences in test anxiety as a situation-specific personality trait (Spielberger, 1972; Spielberger, Gozalez, Taylor, Ross, & Anton, 1977). The TAI consists of 20 statements pertaining to feelings and reactions while taking tests. Test Anxiety in this study referred to an unpleasant emotional state or feelings which could affect the behavioural, cognitive and physiological aspects of an individual. Scores may range from 20 to 80, and higher scores indicate higher levels of anxiety. The TAI indicates subscales of 8 items each for the Worry and Emotionality components of test anxiety.

The five non-routine mathematical problems in the Problems Test were selected, adapted and modified from various sources such as handbook for elementary school teachers (Krulik & Rudnick, 1998) (see Appendix). These items, non-routine mathematical tasks that engaged the students in work with familiar situations, were not readily solved by learned algorithms or standard procedures but presented situations which required the use of various heuristics. The language of the tasks was appropriate for Secondary 2 students. The Problems Test instrument was validated by a six-member team of mathematics educators.

Procedure
The Fennema-Sherman Mathematics Anxiety Scale (MAS) and the Test Anxiety Inventory (TAI) were administered together one week before the Problems Test. This sequence was selected to avoid providing the students with clues and recall of the anxiety that may have been aroused during the Problems Test. In the administration of the MAS and TAI, the teachers referred to the MAS as the Mathematics Attitudes Scale and the TAI as the Test Attitude Inventory, as printed on both forms and the term "anxiety" was avoided. The mathematics teachers were not present in class during the survey. The students were given enough time to respond to all the items. Though no time limit was set, the students completed the

two questionnaires within 30 minutes. As the instructions and statements were clear, no questions were asked during the course of the administration of the questionnaires. The Problems Test was administered one week after the students responded to the Fennema-Sherman Mathematics Anxiety Scale (MAS) and the Test Anxiety Inventory (TAI). The Problems Test was administered under examination conditions. The students were not told in advance about the tests. They were given sufficient time to complete the tests. The Problems Test has a maximum score of 20 marks; that is, 4 marks each for each item. The Problems Test was scored using a focused holistic scoring scheme adapted from Charles, Lester, and O'Daffer (1987).

Descriptive statistics and correlation analysis were obtained using SPSS (Version 11) after the collation of the data. The means and standard deviations for the questionnaires and the Problems Test were calculated. A further analysis of Problems Test scores was made by comparing five groups of students of different mathematics-anxiety levels from their Mathematics Anxiety Scale scores. Each level comprised approximately 20 per cent of the total sample, with the lowest level of mathematics anxiety was designated as level 1 through to the highest at level 5. A Pearson r, product-moment-correlation analysis was conducted. A one-way-analysis of variance (ANOVA) was carried out for the Problems Test on the five mathematics-anxiety levels.

Results and Discussions

The means and standard deviations were computed for the Fennema-Sherman Mathematics Anxiety Scale, Test Anxiety Inventory (TAI), and Problems Test. Table 1 gives a summary of the means and standard deviations of scores for the

Table 1
Mean Scores and Standard Deviations (SD) for the Mathematics Anxiety, Test Anxiety, and Problems Test

Variables	Mean	S.D
Mathematics Anxiety (max = 60)	33.27	10.45
Test Anxiety (max = 80)	43.50	10.30
Problems Test (max = 20)	14.18	3.59

three variables: Mathematics Anxiety, Test Anxiety, and Problems Test for the sample of 621 Secondary 2 students. Scores on the mathematics anxiety scale show a mean of 33.27 and a standard deviation of 10.45, while the test anxiety scores gave a mean of 43.50 and a standard deviation of 10.30. The mean score of 14.18 for the Problems Test scores might indicate that most of these Secondary 2 students in the sample were slightly above average in their problem-solving performance, if the generally accepted passing mark of 10 was taken as the criterion.

Results of the correlation analysis, in this case the Pearson correlation, are presented in Table 2. Table 2 provides a breakdown of the correlations among scores of the students on the Mathematics Anxiety, Test Anxiety, and Problems Test Scores and shows that most of the correlations are statistically significant. However, the Problems Test score did not correlate significantly with the test anxiety indicating that there was no relationship between problems-solving ability and test anxiety. This suggests that students with low test anxiety will not necessarily do better than the high test anxious students while solving non-routine problems. The relationship between mathematics anxiety and test anxiety was found to be moderately positively related ($r = 0.39$, $p < 0.005$). That is, there was a correlation between the two variables, with high levels of mathematics anxiety associated with high levels of test anxiety. These results indicated that high test-anxious students exhibited high levels of mathematics anxiety more frequently than the low-test anxious students. Correlation analysis showed a moderately negative relationship between Problems Test scores and mathematics-anxiety scores of the students ($r = -0.20$, $p < 0.005$). Although the Problems Test scores were negatively correlated ($p < 0.005$) with the mathematics-anxiety scores, the correlation was weak thus partly supporting the idea that higher mathematics anxiety is associated with lower Problems Test scores but lower mathematics anxiety does not necessarily relate to optimal problem-solving abilities.

Table 2
Correlations among Mathematics Anxiety, Test Anxiety, and Problems Test Scores

Scale	2	3
1. Mathematics Anxiety	0.39*	-0.20*
2. Test Anxiety		-0.04 (NS)
3. Problems Test		-

* Correlations are statistically significant at the 0.005 level (2-tailed)

Table 3 shows the Problems Test mean scores and Test Anxiety mean scores at each level of mathematics anxiety.

Table 3
Mean Scores and Standard Deviations (SD) for the Problems Test Scores and Test Anxiety Mean Scores at Five Levels of Mathematics Anxiety

Levels of Mathematics Anxiety	Problems Test (max = 20)		Test Anxiety (max = 80)	
	Mean	S.D.	Mean	S.D.
Level 1 (N = 131)	15.53	3.40	39.22	9.83
Level 2 (N = 126)	14.29	3.77	40.74	8.36
Level 3 (N = 115)	14.05	3.32	42.42	10.00
Level 4 (N = 117)	13.51	3.13	45.50	9.63
Level 5 (N = 132)	13.42	3.87	49.55	10.17

The mean scores for the Problems Test ranged from 13.42 to 15.53 marks while the test anxiety ranged from 39.22 to 49.55. These results were not surprising as many teachers and researchers associate higher levels of mathematics anxiety with lower problem-solving success and higher subject-specific manifestation of test anxiety (Bandalos, Yates, & Thorndike-Christ, 1995; Brush, 1981; Hembree, 1990).

To confirm the results shown in Table 3 that there was a decrease in the means of the Problems Test scores through the mathematics-anxiety levels, a one-way ANOVA for the Problem Test scores was carried out at a five per cent level of significance. This analysis of the Problems Test scores (F = 7.48, p= 0.000) showed that the differences between the mean scores of the five mathematics-anxiety levels were statistically significant. The relationship depicted in Figure 1 shows that as the mathematics anxiety level increased, the mean scores of the students' Problems Test

decreased; that is, as students became increasingly more anxious, they scored lower in their Problems Test. On the whole, the relationship suggested that low mathematics-anxious students performed better in non-routine problems than the high mathematics-anxious students.
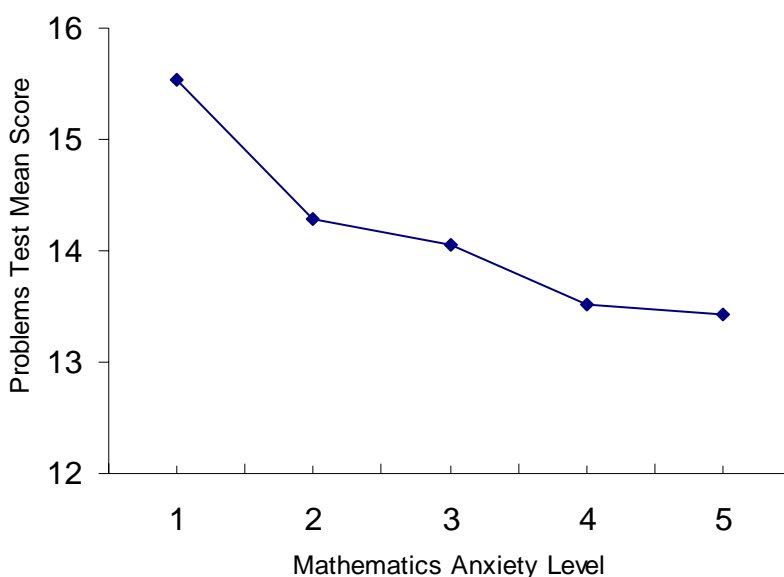

Figure 1. Problems Test Mean Scores against Levels of Mathematics Anxiety

The results of the one-way ANOVA, while statistically significant, do not really tell anything more than there is at least one difference somewhere within the five means. A post-hoc pair-wise test between the means for each mathematics anxiety level was thus conducted to observe any differences among the five means. Table 4 shows the Tukey's honesty-significant-difference test for the five mean differences at each level of mathematics anxiety.

Post-hoc comparisons using the Tukey HSD test indicated that the Problems Test mean score for mathematics-anxiety level 1 (lowest mathematics-anxiety level) was significantly different from mathematics-anxiety levels 2, 3, 4, and 5. However, there was no significant difference observed among the Problems Test mean scores for the other mathematics-anxiety levels (see Table 4). Thus, students with the lowest mathematics-anxiety level performed significantly better than the students in the other four mathematics-anxiety levels. This was not surprising as the Fennema-

Sherman Mathematics Anxiety Scale used in this study measured secondary school students' emotional state in mathematical situations such as lessons in class, homework and solving difficult mathematics problems. The mathematics-anxious students experienced more frequently higher levels of worry and emotionality especially during mathematical situations which prevented them from giving full attention to the task at hand while the low mathematics-anxiety students were not distracted by such task-irrelevant and self-deprecatory reactions and proceeded straight to the problems at hand. This observation is in agreement with the views suggested by Buxton (1981), Lester (1983), Mandler (1989), Schoenfeld (1989), and Trimmer (1974).

Table 4
Tukey's Honesty Significant Difference Test for the Problems Test Scores at Five
Levels of Mathematics Anxiety

	Level 1 (N = 131)	Level 2 (N = 126)	Level 3 (N = 115)	Level 4 (N = 117)	Level 5 (N = 132)
Level 1		1.25*	1.48*	2.02*	2.11*
Level 2			0.23	0.77	0.86
Level 3				0.54	0.63
Level 4					0.09

* The mean difference is significant at the 0.05 level.

In summary, the findings showed a positive relationship between mathematics anxiety and test anxiety. In addition, the study also indicated that mathematics anxiety and performance on a non-routine mathematical problem-solving test showed a marginal linear relationship whereas test anxiety had almost no relationship with the performance on a non-routine mathematical problem solving test.

Discussion and Conclusion

The evidence of the existence of a relationship between mathematics anxiety and mathematical problem-solving performance indicates that attention should be given to the mathematics anxiety phenomenon. There is a need to explore the possible causes of mathematics anxiety especially among Secondary 2 students in Singapore.

It is possible that mathematics anxiety could be caused by students' lack of confidence in their ability to solve non-routine mathematical problems, or a lack of exposure to problem-solving heuristics. In particular, it is important for classroom teachers to have an awareness of the kind of remedial work and individual help and counseling that can assist students to restore their confidence in solving non-routine problems. Classroom teachers could also explore other ways of monitoring and evaluating students' progress. The scope of the problem-solving test, the level of difficulty and the time allocated are important considerations. With regard to the interaction between anxiety and performance in mathematical problem solving, this co-relational study however provides no basis with respect to cause or effect. Although some educators may hypothesize that under-achievement may be due to factors like "poor teaching causes mathematics anxiety in students," there is no conclusive evidence from this study to show this is so.

Although this study represents an initial exploration into the interrelationships among mathematics anxiety, test anxiety, and problem-solving performance of Secondary 2 students in Singapore, the findings of this study provide evidence that mathematics anxiety and mathematical problem solving are related while test anxiety does not appear to have any association with mathematical problem solving. This represents a further contribution to research on secondary students' mathematics anxiety and mathematical problem solving as well as providing a way forward in understanding how mathematics anxiety can affect the solving of non-routine problems.

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Author:

Yeo Kai Kow, Joseph. Assistant professor, National Institute of Education, Nanyang Technological University, Singapore. kkyeo@nie.edu.sg

Appendix
Problems Test

1. John bought 12 stamps of denominations 15 cents, 20 cents and 50 cents. The total cost of the stamp is $2.90. How many of each kind of stamps would he buy so that it makes up a total cost of $2.90? Give all possible sets of answers.
Show all your workings and explain it.

2. There were 8 students in a tennis tournament. If each student played each other person one time, how many matches would have to be played to determine a winner?
Explain how you worked it out.

3. A particular car park is only allowed to park cars and motorcycles. A count shows that there are 45 vehicles and 150 wheels in this car park when it is full.
How many motorcycles are there when the car park is full?
Explain how you worked it out.

4. A Mathematics quiz consists of 20 multiple-choice questions. A correct answer is awarded 5 marks and 2 marks are deducted for a wrong answer while no marks are awarded or deducted for each question left unanswered. If a boy scores 48 marks in the quiz, what is the *greatest* possible number of questions he answered correctly?
Explain how you worked it out

5. Peter loaned $7 to John. Peter borrowed $15 from Robert and $32 from Kelvin. Moreover, Kelvin owes $3 to Robert and $7 to John. One day the boys decided to get together to sort out their accounts. Which boy left with $18 more than he came with?
Show all your workings and explain it.